4.1 Is There a Pattern Here?

Recognizing Patterns and Sequences

LEARNING GOALS

In this lesson, you will:

- Recognize patterns.
- Describe patterns.
- Represent patterns as sequences.
- Predict the next term in a sequence.

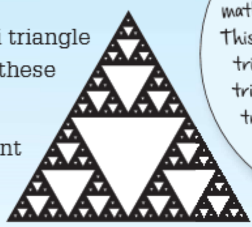
KEY TERMS

- sequence
- term of a sequence
- infinite sequence
- finite sequence

Want to play the chaos game? Dr. Michael Barnsley coined the phrase "chaos game," in which a pattern can be created by plotting a random point within a triangle and then rolling a number cube. A few hundred rolls of the number cube result in a pattern that forms what is called the Sierpinski triangle.

The Sierpinski triangle is named after the Polish mathematician Waclaw Sierpinski. This triangle consists of one large triangle, and within the larger triangle consists three smaller triangles, each of which also contain three smaller triangles, and so on.

Care to see if you can create a Sierpinkski triangle by merely rolling a number cube? Follow these steps to see if you can do it.



1. First plot three points that will represent the vertices of a triangle. Label these points *A*, *B*, and *C*.
2. Plot a point anywhere inside the triangle.
3. Next, roll the number cube. If you roll a 1 or 2, measure half the distance from your initial point to vertex *A* and plot this point. If you roll a 3 or 4, measure half the distance from your initial point to vertex *B* and plot the point. If you roll a 5 or 6, measure half the distance from your initial point to vertex *C* and plot the point.
4. Repeat the process in Step 3, but this time, start with your new plotted point. Do this a few hundred times, and you may begin to create a Sierpinski triangle. Good luck!

PROBLEM 1 Do You See a Pattern?

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term of a sequence** is an individual number, figure, or letter in the sequence.

Examples of sequences are shown. Describe the pattern, draw or describe the next terms, and represent each sequence numerically.

"Positive Thinking"

- Analyze the number of dots. Describe the pattern.
- Draw the next three figures of the pattern.

4

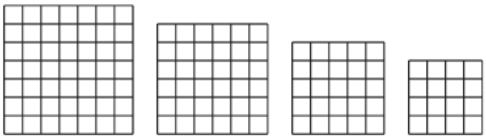
- Write the sequence numerically to represent the number of dots in each of the first 7 figures.

Family Tree

Jessica is investigating her family tree by researching each generation, or set, of parents. She learns all she can about the first four generations, which include her two parents, her parents' parents, her parents' parents' parents, and her parents' parents' parents' parents.

- Think about the number of parents. Describe the pattern.
- Determine the number of parents in the fifth and sixth generations.
- Write a numeric sequence to represent the number of parents in each of the 6 generations.

A Collection of Squares



- Analyze the number of small squares in each figure. Describe the pattern.

- Draw the next three figures of the pattern.

- Write the sequence numerically to represent the number of small squares in each of the first 7 figures.

4

Al's Omelets

Al's House of Eggs N'at makes omelets. Al begins each day with 150 eggs to make his famous *Bestern Western Omelets*. After making 1 omelet, he has 144 eggs left. After making 2 omelets, he has 138 eggs left. After making 3 omelets, he has 132 eggs left.

- Think about the number of eggs Al has left after making each omelet. Describe the pattern.

- Determine the number of eggs left after Al makes the next two omelets.

- Write the sequence numerically to represent the number of eggs left after Al makes each of the first 5 omelets. Include the number of eggs he started with.

Mario's Mosaic

Mario is creating a square mosaic in the school courtyard as part of his next art project. He begins the mosaic with a single square tile. Then he adds to the single square tile to create a second square made up of 4 tiles. The third square he adds is made up of 9 tiles, and the fourth square he adds is made up of 16 tiles.

- Think about the number of tiles in each square. Describe the pattern.

- Determine the number of tiles in the next two squares.

- Write the sequence numerically to represent the number of tiles in each of the first 6 squares.

Troop of Triangles

4

- Analyze the number of dark triangles. Describe the pattern.

- Draw the next two figures of the pattern.

- Write the sequence numerically to represent the number of dark triangles in each of the first 6 figures.

Gamer Guru

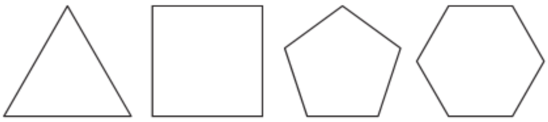
Mica is trying to beat his high score on his favorite video game. He unlocks some special mini-games where he earns points for each one he completes. Before he begins playing the mini-games (0 mini-games completed), Mica has 500 points. After completing 1 mini-game he has a total of 550 points, after completing 2 mini-games he has 600 points, and after completing 3 mini-games he has 650 points.

- Think about the total number of points Mica gains from mini-games. Describe the pattern.

- Determine Mica's total points after he plays the next two mini-games.

- Write the sequence numerically to represent Mica's total points after completing each of the first 5 mini-games. Include the number of points he started with.

Polygon Party



4

- Analyze the number of sides in each polygon. Describe the pattern.

- Draw the next two figures of the pattern.

- Write the sequence numerically to represent the number of sides of each of the first 6 polygons.

Pizza Contest

Jacob is participating in a pizza-making contest. Each contestant not only has to bake a delicious pizza, but they have to make the largest pizza they can. Jacob's pizza has a 6-foot diameter! After the contest, he plans to cut the pizza so that he can pass the slices out to share. He begins with 1 whole pizza. Then, he cuts it in half. After that, he cuts each of those slices in half. Then he cuts each of those slices in half, and so on.

- Think about the size of each slice in relation to the whole pizza. Describe the pattern.
- Determine the size of each slice compared to the original after the next two cuts.
- Write the sequence numerically to represent the size of each slice compared to the original after each of the first 5 cuts. Include the whole pizza before any cuts.

Coin Collecting

4

Miranda's uncle collects rare coins. He recently purchased an especially rare coin for $5. He claims that the value of the coin will triple each year. So even though the coin is currently worth $5, next year it will be worth $15. In 2 years it will be worth $45, and in 3 years it will be worth $135.

- Think about how the coin value changes each year. Describe the pattern.
- Determine the coin value after 4 years and after 5 years.
- Write the sequence numerically to represent the value of the coin after each of the first 5 years. Include the current value.

PROBLEM 2 What Do You Notice?

There are many different patterns that can generate a sequence of numbers. For example, you may have noticed that some of the sequences in Problem 1, *Do You See a Pattern?* were generated by performing the same operation using a constant number. In other sequences, you may have noticed a different pattern.

The next term in a sequence is calculated by determining the pattern of the sequence, and then using that pattern on the last known term of the sequence.

1. For each sequence in Problem 1, write the problem name and numeric sequence in the table shown. Also in the table, record whether the sequence increases or decreases, and describe the operation(s) used to create each sequence. The first one has been done for you.

Problem Name	Numeric Sequence	Increases or Decreases	Sequence Description
"Positive Thinking"	25, 21, 17, 13, 9, 5, 1	Decreases	Begin at 25. Subtract 4 from each term.

4

2. Which sequences are similar? Explain your reasoning.

PROBLEM 3 Do Sequences *Ever* End?

1. Consider a sequence in which the first term is 64, and each term after that is calculated by dividing the previous term by 4.

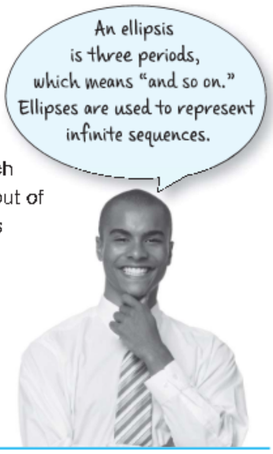
   Margaret says that this sequence ends at 1 because there are no whole numbers that come after 1. Jasmine disagrees and says that the sequence continues beyond 1. Who is correct? If Margaret is correct, explain why. If Jasmine is correct, predict the next two terms of the sequence.

If a sequence continues on forever, it is called an **infinite sequence**. If a sequence terminates, it is called a **finite sequence**.

For example, consider an auditorium where the seats are arranged according to a specific pattern. There are 22 seats in the first row, 26 seats in the second row, 30 seats in the third row, and so on. Numerically, the sequence is 22, 26, 30, . . . , which continues infinitely. However, in the context of the problem, it does not make sense for the number of seats in each row to increase infinitely. Eventually, the auditorium would run out of space! Suppose that this auditorium can hold a total of 10 rows of seats. The correct sequence for this problem situation is:

22, 26, 30, 34, 38, 42, 46, 50, 54, 58.

Therefore, because of the problem situation, the sequence is a finite sequence.

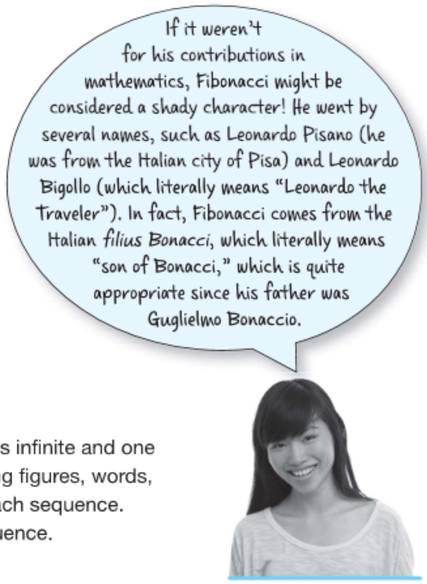


2. Does the pattern shown represent an infinite or finite sequence? Explain your reasoning.

3. One of the most famous infinite sequences is the Fibonacci sequence. The first 9 terms in the Fibonacci sequence are shown:

0, 1, 1, 2, 3, 5, 8, 13, 21, . . .

Explain in your own words the pattern that determines the Fibonacci sequence. Then, predict the next five terms in the sequence.



4

4. Write your own two sequences—one that is infinite and one that is finite. Describe your sequence using figures, words, or numbers. Give the first four terms of each sequence. Explain how you know that each is a sequence.

Be prepared to share your solutions and methods.